Time-Frequency Analysis for Nonstationary Random Response of Vehicle

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In this paper the nonstationary random response of braking vehicle in time domain was first obtained by using nonstationary road roughness model and vehicle model with 5DOF. To obtain the result of nonstationary response in frequency domain, the maximum entropy method which had proved to have more advantages over traditional FFT method was used for processing nonstationary response of braking vehicle in frequency domain. Finally, the three-dimensional transient maximum entropy spectrum (MES) of response was given.

1 Introduction

It is known that most research works on Random vibration of vehicle caused by the excitation of rough road in frequency domain was based on two approaches. One approach is direct method in which frequency characteristics can be obtained directly by using transfer function according to the theory of random vibration (Dokainish, M.A. and Elmadany, M.M, 1980). The other is an indirect method in which the response of vehicle in time domain is first obtained, and then the response of vehicle in frequency domain can be obtained by using FFT method which is widely used for processing stationary signal (Brigham, E.O.1988; Welch, P.D, 1967). While a car is traveling at variable speed such as starting, accelerating as well as braking, the function of road roughness is a non-stationary random process in time domain despite it is stationary random process in spatial domain. Therefore the response of vehicle to the excitation of rough road should be nonstationary random process. It is not proper to use FFT to process input and response of braking vehicle, because FFT method is just suitable for processing stationary signals. In this paper, maximum entropy method in which contains more information with less data is used to obtain transient frequency characteristics. It is necessary to point out that the nonstationary response of vehicle in time domain should be given a briefly statement first.

2 Nonstationary Response of Vehicle

In general case, the function of road roughness is regarded as a stationary random process in space domain. As a car is moving with constant velocity, it is also a stationary random process in time domain. However road roughness is a non-stationary random process in time domain while a vehicle is traveling at variable speed. Therefore the vibration caused by rough road surface should be also considered as a non-stationary random process. A state-space approach was presented to analyze the response of a vehicle traveling on homogenous rough road (Hammond and Harrison, 1981). In that work, the dynamics were modeled by liner ordinary differential equations in time domain while the excitation process was modeled by a differential equation in spatial domain. The variance of response was obtained by using so-called covariance equivalent modeling. Based on this work, an improved method with more computational efficiency was proposed by using complex modal analysis (Hwang, J. H. and Kim, J. S, 2000). There was another method for solving this problem (Nigam and Yaday, 1974; Fang, T., 1997), in which differential equations with variable coefficient were established first in space domain, and then, the time changing covariance was computed. However, the amount of computation of this method is large. This section mainly investigates a new time method by using the nonstationary excitation model of a rough road.

2.1 Modeling

In this section, a new method of solving non-stationary vibration of a vehicle is mainly investigated, which can be also used in various kinds of vehicle models. Therefore, as an example, a vehicle model with 5 degrees of freedom (DOF) is established. The vehicle model is shown in **Figure 1**.

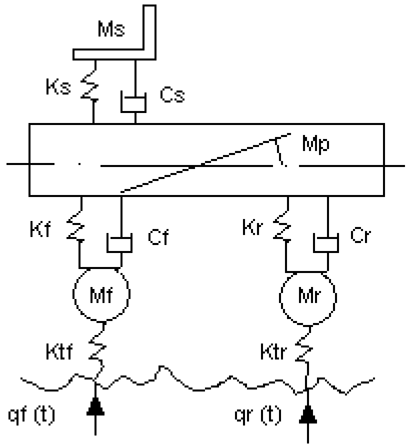

Fig. 1 Vehicle model with 5 degrees of freedom

Where:

- Z_s-vertical displacement of seat,
- Z_b-vertical displacement of vehicle body at center of gravity,
- Z_p-pitch angular displacement of vehicle body,
- Z_f-bouncing displacement of unsprung mass of front suspension,
- M_s-mass of human body and seat,
- M_b-mass of vehicle body,
- M_p-moment of inertia of vehicle body around y axis,
- K_s-stiffness coefficient of seat,
- M_f, M_r-unsprung masses of front and rear suspensions, respectively,
- K_f, K_r-stiffness coefficients of front and rear suspensions, respectively,
- K_{tf}, K_{tr}-stiffness coefficients of front and rear tires, respectively,
- C_s-damping coefficient of seat,
- C_f, C_r-damping coefficients of front and rear suspensions, respectively,
- $q_f(t)$, $q_r(t)$-displacements of road input at front and rear tires, respectively,
- l_1-distance from the center of gravity to seat,
- l_2, l_3-distances from the center of gravity to front and rear tires, respectively,
- L-wheelbase.

In this model, damping of tires is neglected, since it is so small. The nonlinear damping and stiffness of suspension can be also considered if necessary.

The differential equations for the vehicle model can be described as follows:

$$[M]\ddot{Z} + [C]\dot{Z} + [K]Z = [F]Q \tag{1}$$

where:

Z-output vector

$$Z = \begin{bmatrix} z_s & z_b & z_p & z_f & z_r \end{bmatrix}^T$$

$[M]$-matrix of mass

$$[M] = \begin{bmatrix} M_s & 0 & 0 & 0 & 0 \\ 0 & M_b & 0 & 0 & 0 \\ 0 & 0 & M_p & 0 & 0 \\ 0 & 0 & 0 & M_f & 0 \\ 0 & 0 & 0 & 0 & M_r \end{bmatrix}$$

$[C]$-damping matrix

$$[C] = \begin{bmatrix} C_s & -C_s & C_s l_1 & 0 & 0 \\ -C_s & C_s + C_f + C_r & -C_s l_1 - C_f l_2 + C_r l_3 & -C_f & -C_r \\ C_s l_1 & -C_s l_1 - C_f l_2 + C_r l_3 & C_s l_1^2 + C_f l_2^2 + C_r l_3^2 & C_f l_2 & -C_r l_3 \\ 0 & -C_f & C_f l_2 & C_f & 0 \\ 0 & -C_r & -C_r l_3 & 0 & C_r \end{bmatrix}$$

$[K]$-stiffness matrix

$$[K] = \begin{bmatrix} K_s & -K_s & l_1 K_s & -K_f & 0 \\ -K_s & K_s + K_f + K_r & -K_s l_1 - K_f l_2 + K_r l_3 & K_f l_2 & -K_r \\ l_1 K_s & -K_s l_1 - K_f l_1 + K_r l_3 & K_s l_1^2 + K_f l_2^2 + K_3 l_3^2 & K_f + K l_2 & -K_r l_3 \\ 0 & -K_f & K_f l_2 & K_f + K_{tf} & 0 \\ 0 & K_r & -K_r l_3 & 0 & K_r + K_{tr} \end{bmatrix}$$

$[F]$-excitation force matrix

$$[F] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_{tf} & 0 \\ 0 & K_{tr} \end{bmatrix}$$

$[Q]$-excitation vector

$$[Q] = \begin{bmatrix} q_f & q_r \end{bmatrix}^T$$

When a car is running at variable speed, functions of $q_f(t)$ and $q_r(t)$ are non-stationary in time domain, but it is noted that they are stationary in spatial domain.

2.2 State Equations

To obtain the results of equation (1) in time domain, state vectors are utilized as follows:

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T = \begin{bmatrix} z_s & z_b & z_p & z_f & z_r \end{bmatrix} \tag{1-a}$$

$$\begin{bmatrix} x_6 & x_7 & x_8 & x_9 & x_{10} \end{bmatrix}^T = \begin{bmatrix} \dot{z}_s & \dot{z}_b & \dot{z}_p & \dot{z}_f & \dot{z}_r \end{bmatrix} \tag{1-b}$$

Hence

$$\begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \dot{x}_3 & \dot{x}_4 & \dot{x}_5 \end{bmatrix}^T = \begin{bmatrix} x_6 & x_7 & x_8 & x_9 & x_{10} \end{bmatrix} \tag{1-c}$$

$$\begin{bmatrix} \dot{x}_6 & \dot{x}_7 & \dot{x}_8 & \dot{x}_9 & \dot{x}_{10} \end{bmatrix}^T = \begin{bmatrix} \ddot{z}_s & \ddot{z}_b & \ddot{z}_p & \ddot{z}_f & \ddot{z}_r \end{bmatrix} \tag{1-d}$$

Substituting these state vectors into Eq. (1), then the following state equation can be obtained:

$$\dot{X} = AX + BU \tag{2}$$

where:

X-state variables

$$X = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \end{bmatrix}^T$$

2.3 Simulation of Road Roughness

In general case, the power spectral density (PSD) of rough road in frequency domain can be expressed as:

$$S_q(\omega) = S_q(\Omega_0) v / (\omega^2 + {\omega_0}^2) \tag{3}$$

where, $S_q(\Omega_0)$ is the coefficient of road roughness, According to the classification of road, the values of $S_q(\Omega_0)$ can be obtained from some references. Ω_0 is the reference value of spatial angular frequency, $\Omega_0 = 1(rad/m)$, v is traveling speed of vehicle, and ω_0 is the lowest cut-off angular frequency. Eq.(3) can be considered as a response of a first order linear system to white noise excitation. Based on the theory of random vibration, following relationship is obtained

$$S_q(\omega) = |H(\omega)|^2 S_w \tag{4}$$

where $H(\omega)$ is the transfer function, and S_w is the PSD of white noise where normally $S_w = 1$. From Eqs (3) and (4), $H(\omega)$ is written by

$$H(\omega) = \frac{\sqrt{S_q(\Omega_0)v}}{\omega_0 + j\omega} \tag{5}$$

From Eq.(5), the differential equation about road roughness is expressed as

$$\dot{q}(t) + \omega_0 q(t) = \sqrt{S_q(\Omega_0)v}\,w(t) \tag{6}$$

While a car is running with variable speed, despite road roughness is stationary random process in space domain, it is non-stationary random process in time domain. Note that

$$\omega = 2\pi n v(t) \tag{7}$$

where n is space frequency. Therefore, Eq.(6) becomes as follows

$$\dot{q}(t) + 2\pi n_0 v(t) q(t) = 2\pi\sqrt{S_q(n_0) n_0^{\,2} v(t)}\,w(t) \tag{8}$$

where $v(t) = v_0 + at$, a is the deceleration of car braking, t is braking time.

According to Eq.(8), the numerical simulation can be carried out. The **Figure 2** and **Figure 3** represent the simulation consequence of nonstationary rough road and its PSD respectively. From **Figure 2,** it is concluded that with inreasing of time, the amplitude of road roughness in time domain decreses in both low frequency and high frequency.

2.4 Simulation of Nonstationary Responses

Inputting the values of road roughness into Eq.(2), the non-stationary responses of the vehicle system to excitation of road roughness can be calculated. The model parameters representing 5 degrees of freedom are shown in **Table 1**. **Figure 4** represents the responses of vehicle system to non-stationary excitation, i.e., the responses of vehicle braking with a=-3m/s V_0=60km/h. It has been seen that at the beginning of braking, the vibration accelerations of seat and vehicle body, relative displacement and pitch acceleration are all random processes. When the braking time is at about t=6s, i.e., the velocity of vehicle is equal to zero, All the responses of vehicle become free vibration, which can clearly reflects the natural frequencies of response of vehicle.

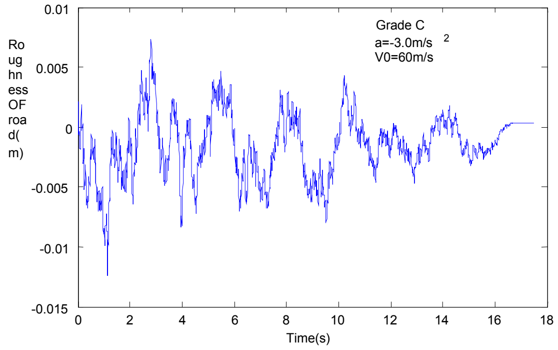

Fig. 2 Non-stationary road roughness

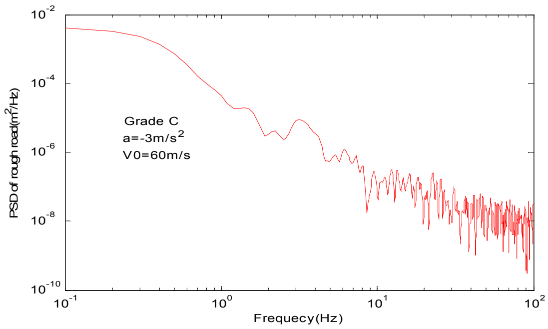

Fig. 3 PSD of nonstationary road

Ms (kg)	Mb (kg)	Mp (kg.m2)	Mf (kg)	Mr (kg)
70	2100	3500	140	210
Ks (N/m)	Kf (N/m)	Kr (N/m)	Ktf (N/m)	Ktr (N/m)
12200	74000	120000	520000	520000
Cs (Ns/m)	Cf (Ns/m)	Cr (Ns/m)		
550	1800	1200		

Table 1 Parameters of vehicle model with 5 DOF

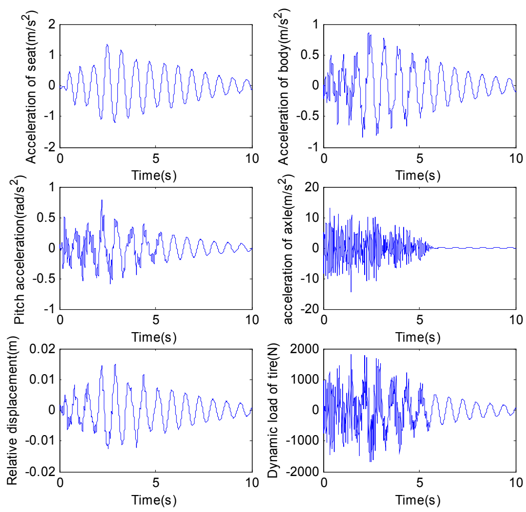

Fig. 4 Nonstationary response of vehicle (V_0=60km/h, a=-3.0m/s^2)

3 Maximum Entropy Spectral Analysis

Maximum entropy spectral analysis was first proposed by Burg in 1967, which has developed in the past twenty years. It is also called modern spectrum compared with classical FFT method. Although there were much more successful applications of FFT to processing stationary signals, the frequency resolution Δf is conflict with sampling time interval Δt in using FFT method. Furthermore the assumption that there is no data beyond the sampling time length T results in reduction of frequency resolution Δf. Therefore it is not suitable to use FFT method to process short time data. Maximum entropy spectral analysis is parameter estimation method in which a data model is first established and then the estimation of PSD is carried out with model parameters. Therefore the frequency resolution is not limited by length of data, and higher frequency distinguishing ability can be obtained.

3.1 Modeling and Parameters Estimation

Given sampled discrete series $x(n)$, where n is sampling number, n=1, 2 ,3, ⋯N. Assuming that $x(n)$ is the response of a linear model subjected to excitation of white noise. Therefore it can be expressed as:

$$x(n) = -\sum_{k=1}^{p} a_k x(n-k) + w(n) \tag{9}$$

where $w(n)$ is white noise with zero mean and $\sigma^2{}_w$ variance, p is the number of model orders, and a_k is model parameters, $k = 1, 2, 3\,p$. According to the definition of autocorrelation

$$R_x(m) = E[x(n)x(n+m)] \tag{10}$$

and considering Eq.(9), the following equation is obtained

$$R_x(m) = E\left[x(n)\left\{-\sum_{k=1}^{p} a_k x(n-k+m) + w(n+m)\right\}\right] \tag{11}$$

Simplifying Eq. (11), and it can be expressed as

$$R_x(m) = -\sum_{k=1}^{p} a_k R_x(m-k) + E[x(n)w((n+m)] \tag{12}$$

when $m \geq 1$ $x(n)$ is irrelevant to $w(n+m)$

Hence

$$E[x(n)w(n+m)] = \begin{cases} 0(m>0) \\ \sigma_w{}^2(m=0) \end{cases} \tag{13}$$

Substituting Eq.(13) into Eq.(12), the following equations are obtained.

$$\begin{cases} R_x(m) + \sum_{k=1}^{p} a_k R_x(m-k) = 0(m>0) \\ R_x(0) + \sum_{k=0}^{p} a_k R_x(-k) = \sigma_w{}^2(m=0) \end{cases} \tag{14}$$

Eq.(14) can also be written in terms of matrix

$$\begin{bmatrix} R_x(0) & R_x(-1) & \cdots & R_x(-p) \\ R_x(1) & R_x(0) & \cdots & R_x(-p+1) \\ \vdots & \vdots & \vdots & \vdots \\ R_x(p) & R_x(p-1) & \ldots & R_x(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \sigma_w{}^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{15}$$

While given sampled discrete series $x(n)$, autocorrelation R_x and covariance $\sigma_w{}^2$ can be calculated. According to Eq.(15), model parameters can be obtained. So far there are many algorithms for computing model parameters among which Burg algorithm is a fast algorithm similar to FFT. While more data need to be calculated, this method is much effective.

3.2 Burg Fast Algorithm

Burg fast algorithm can obtain the model parameters directly with sampled data without calculating the autocorrelation. It is noted that the values obtained from Eq.(9) are the evaluated values of sampled data. Define $\tilde{x(n)}$ as the evaluated values. Therefore, the forward error of data model with p orders is expressed as:

$$e_{fn}^{(p)} = x(n) - \tilde{x(n)} = x(n) + \sum_{k=1}^{p} a_k^{(p)} x(n-k) \tag{16}$$

Similarly, the backward error of data model with p orders is written by

$$e_{bn}^{(p)} = x(n) - x(\tilde{n-p}) = x(n) + \sum_{k=1}^{p} a_k^{(p)} x(n+k-p) \tag{17}$$

The relationship between high order error and low order error can be expressed as

$$e_{fn}^{(p)} = e_{fn}^{(p-1)} + \rho_p e_{bn-1}^{(p-1)} \tag{18}$$

$$e_{bn}^{(p)} = e_{bn}^{(p-1)} + \rho_p e_{bn}^{(p-1)} \tag{19}$$

where ρ_p is the reflecting coefficient. Define ES as the sum of error square.

Hence

$$ES = \sum_{n=p}^{N-1} \left\{ \left[e_{fn}^{(p)}\right]^2 + \left[e_{bn}^{(p)}\right]^2 \right\} \tag{20}$$

Let $\left(\dfrac{\partial E}{\partial \rho_p}\right)^p = 0$ (21)

hence

$$\rho_p = -\frac{2E\left[e_{fn}^{(p-1)} e_{bn-1}^{(n-1)}\right]}{E\left[e_{fn}^{(p-1)}\right]^2 + E\left[e_{bn-1}^{(p-1)}\right]^2} \tag{22}$$

The relationship between model parameters and reflecting coefficients can be expressed as:

$$a_p^{(p)} = \rho_p \tag{23}$$

$$a_k^{(p)} = a_k^{(p-1)} + \rho_p a_{p-k}^{(p-1)} \qquad k=1,2,3...p \tag{24}$$

According to Eq.(22) and Eq.(24), the recursive calculation can be accomplished.

3.3 Maximum Entropy Spectral Estimation

Taking Z transform to both sides of Eq.(9), the discrete transfer function is expressed as

$$H(z) = \frac{X(z)}{W(z)} = \frac{1}{A(z)} = \frac{1}{1 + \sum_{k=1}^{p} a_k z^{-k}} \tag{25}$$

hence the PSD of the signal can be obtained

$$P_x(\omega) = \left|H(e^{-j\omega})\right|^2 \sigma_w{}^2 = \frac{\sigma_w{}^2}{\left|1 + \sum_{k=1}^{p} a_k e^{-jwk}\right|^2} \tag{26}$$

In **Figure 5** is the processing result of vehicle body acceleration by using maximum entropy method with data of 32 points. It is seen that maximum entropy spectral estimation well reflects two main peaks with less sampled data. First peak represents resonant frequency of vehicle body, and second peak is close to the resonant frequency of unsprung mass. Moreover the frequency resolution is continuous. **Figure 6** is the PSD of vehicle body acceleration by using FFT method with same sampled data. It illustrates that FFT method cannot reflect two main peaks, and the frequency resolution is bigger.

$$\Delta f = \frac{1}{T} = \frac{1}{N\Delta t} = \frac{1}{32 \times 0.005} = 6.25Hz$$

Therefore maximum entropy method has more advantage than FFT method in processing short time data. **Figure 7** and **Figure 8** represent MES and PSD of axle acceleration respectively. Data of 2000 points are used in both methods. It concludes that maximum entropy method can give the resonant frequency of axle exactly and FFT method cannot do. **Figure 9, Figure 10** and **Figure 11** are the three dimensional transient maximum entropy spectrums of vehicle body acceleration, axle acceleration and pitch angular acceleration in braking, respectively. From **Figure 9**, it is concluded that with the increasing of time i.e. the decreasing of velocity, the amplitude of MES of vehicle body acceleration has the tendency of reduction, and has some fluctuations which reflect the wave shape of rough road. **Figure 10** illustrates the amplitude of transient MES of axle acceleration decreases with increasing of time. **Figure. 11** shows that transient MES of pitch acceleration has two main peaks in both low and high frequency. The peaks in in low frequency reduces with the increasing of time, but does not always reduce. The peak in high frequency decreases with the increasing of time.

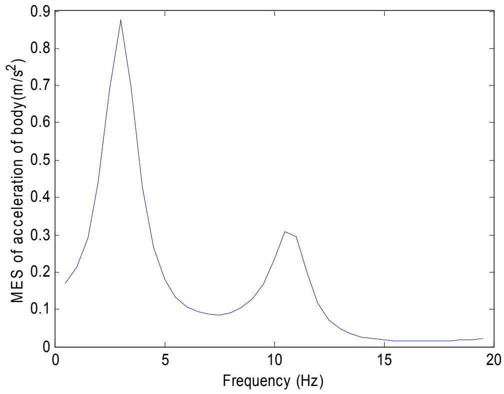

Fig. 5 MES of vehicle body acceleration

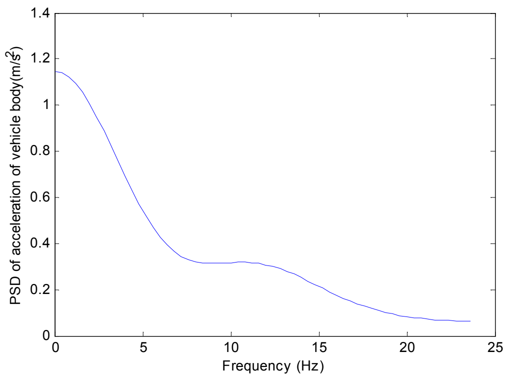

Fig. 6 PSD of vehicle body acceleration

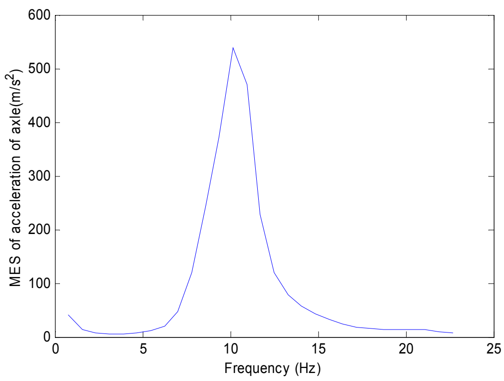

Fig.7 MES of axle acceleration

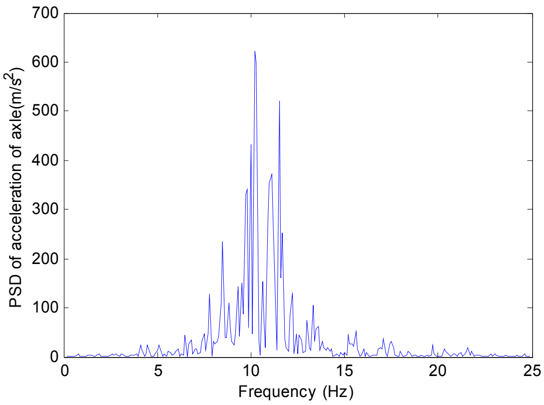

Fig. 8 PSD of axle acceleration

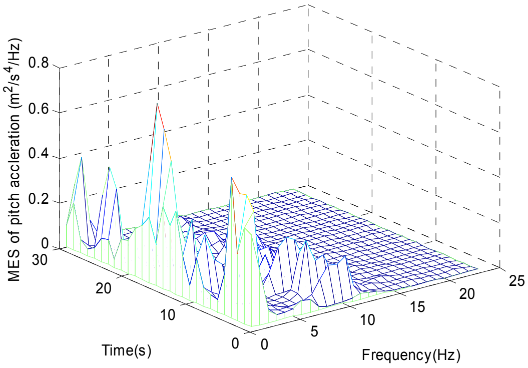

Fig. 11 Transient MES of pitch acceleration

4 CONCLUSIONS

This paper first performs numerical simulation of nonstationary response of vehicle in time domain by using nonstationary excitation model of road and vehicle model with 5DOF. To obtain frequency characteristics, maximum entropy method is used to process nonstationary response in frequency domain. The results show that maximum entropy method has more advantage than traditional FFT method in processing less data. The frequency resolution is successive. The MES method is suitable for processing short time data, therefore the instantaneous spectral characteristics can be obtain, which clearly reflects nonstationary characteristics of vehicle in braking.

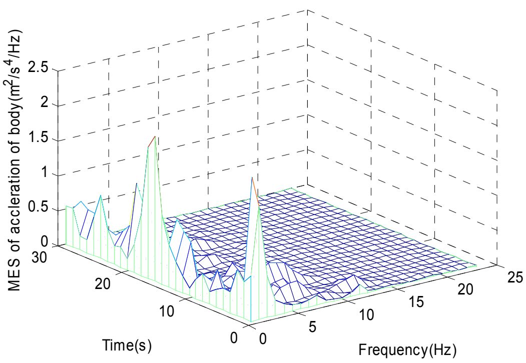

Fig.9 Transient MES of vehicle body acceleration

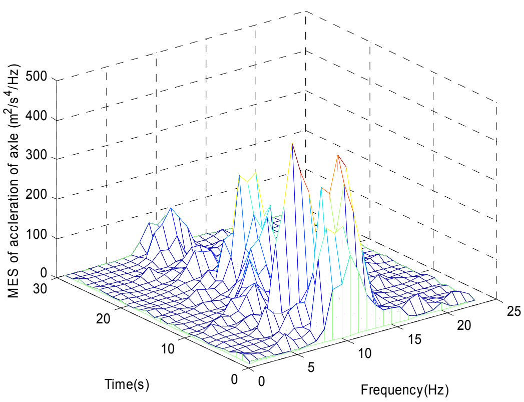

Fig. 10 Transient MES of axle acceleration

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